Level6opaedia

'A level is a level'

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Please note that Using and Applying assessment criteria are not included within the Levelopaedia

Numbers and the Number System

Use the equivalence of fractions, decimals and percentages to compare proportions	
Convert fraction and decimal operators to percentage operators by multiplying by 100. For example: ▪ 0.45; 0.45 × 100% = 45% ▪ 7/12; (7 ÷ 12) × 100% = 58.3% (1 d.p.) Continue to use mental methods for finding percentages of quantities Use written methods, e.g. ▪ Using an equivalent fraction: 13% of 48; 13/100 × 48 = 624/100 = 6.24 ▪ Using an equivalent decimal: 13% of 48; 0.13 × 48 = 6.24	Show me a set of equivalent fractions, decimals and percentages. Show me i) a fraction between 1/3 and ½ ii) a percentage between 1/3 and ½ What is wrong: • 13% of 78 = 78 ÷ 13 = 6 • 40% of 400kg = 400 ÷ 40 = 10 True / Never / Sometimes: To calculate 13% of a quantity, you divide by 13 45s% is greater than 1% Convince me that: • 7/12 = 58.3% (1 d.p.) • 13% of 48 = 6.24

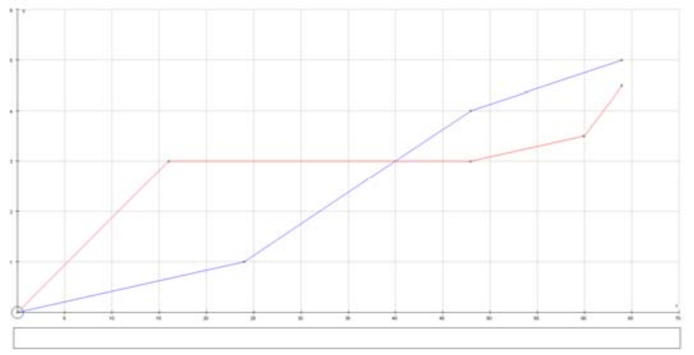
<u>Calculating</u>

Calculate percentages and find the outcome of a given percentage increase or decrease	
Use written methods, e.g. ▪ Using an equivalent fraction: 13% of 48; 13/100 × 48 = 624/100 = 6.24 ▪ Using an equivalent decimal: 13% of 48; 0.13 × 48 = 6.24 ▪ Using a unitary method: 13% of 48; 1% of 48 = 0.48 so 13% of 48 = 0.48 × 13 = 6.24 Find the outcome of a given percentage increase or decrease. e.g. ▪ an increase of 15% on an original cost of £12 gives a new price of £12 × 1.15 = £13.80, ▪ or 15% of £12 = £1.80 £12 + £1.80 = £13.80	Show me an amount and a percentage increase that gives the answer £44 What is the same/different about: • 13/100 × 48 = 624/100 = 6.24 • 0.13 × 48 = 6.24 • 1% of 48 = 0.48 so 13% of 48 = 0.48 × 13 = 6.24 Convince me that £12 increased by 15% is £13.80
Divide a quantity into two or more parts in a given ratio and solve problems involving ratio and direct proportion	
Solve problem such as: ▪ Potting compost is made from loam, peat and sand in the ratio 7:3:2 respectively. A gardener used 1.5 litres of peat to make compost. How much loam did she use? How much sand? ▪ The angles in a triangle are in the ratio 6:5:7. Find the sizes of the three angles.	Show me a quantity divided correctly into a ratio of three parts. Show me how pupils could be in a school if the ratio of pupils with brown hair, blond hair, black hair in a school is 4:2:5. What is the same/different about: 2:7, 3:4:2, 1:4:4 Convince me that if the ratio of pupils in a school with brown hair, blond hair, black hair in a school is 4:2:5 then there cannot be 122 pupils in the school. Convince me that if the ratio of pupils in a school with brown hair, blond hair, black hair in a school is 4:2:5 and there are 24 pupils with blond hair, the number of pupils in the school is 132.
Use proportional reasoning to solve a problem, choosing the correct numbers to take as 100%, or as a whole	
Use unitary methods and multiplicative methods, e.g. ▪ There was a 25% discount in a sale. A boy paid £30 for a pair of jeans in the sale. What was the original price of the jeans? ▪ When heated, a metal bar increases in length from 1.25m to 1.262m. Calculate the percentage increase correct to one decimal place.	Show me a problem you can solve using the unitary method. What is wrong: If a boy paid £30 for a pair of jeans in a 25% discount sale, the original price of the jeans was £37.50. True/never/sometimes: • The inverse of 'increase by 10%' is decrease by 10%' • The inverse of 'increase by 25% is decrease by 20%' Convince me that if a boy paid £30 for a pair of jeans in a 25% discount sale, the original price of the jeans was £40.
Add and subtract fractions by writing them with a common denominator, calculate fractions of quantities (fraction answers); multiply and divide an integer by a fraction	
Add and subtract more complex fractions such as 11/18 + 7/24 , including mixed fractions. Find 2/7 of 5kg. Multiply an integer by a fraction such as 4 x 3/5 Divide an integer by a fraction such as 3 ÷ 2/3	Show me: • a pair of fractions with a sum of 3/5 • a pair of fractions with a difference of 2/7 • a fraction and a quantity such that the answer is 6kg What is the same/different about: • 2/3 + 3/8, 1/12 + 3/4, 5/6 + 1/4

Solve problems involving fractions, e.g. ▪ In a survey of 24 pupils 1/3 liked football best, 1/4 liked basketball, 3/8 liked athletics and the rest liked swimming. How many liked swimming?	• 17/24 – 2/3, 5/8 – 2/3, 11/12 – 3/4 True/never/sometimes: • Pairs of fractions have one common denominator • When adding fractions, both denominators have to be changed • When subtracting fractions, both denominators have to be changed • Multiplying an integer by a fraction gives a smaller answer • Dividing an integer by a fraction gives a smaller answer Convince me: • that 2/7 + 3/5 = 31/35 • that 3 ÷ 2/3 = 4½

Algebra

Use systematic trial and improvement methods and ICT tools to find approximate solutions to harder equations	
Use trial and improvement for algebraic problems, e.g. $x^3 + x = 20$ Use trial and improvement for equivalent problems, e.g. ▪ A number plus its cube is 20, what's the number? ▪ The length of a rectangle is 2cm longer than the width. The area is 67.89cm^2. What's the width? Pupils should have opportunities to use a spreadsheet for trial and improvement methods	Convince me that the solution to 1 decimal place for the equation $x^3 + x = 20$ is x = 2.6
Construct and solve linear equations with integer coefficients, using an appropriate method	
Solve, e.g. ▪ 3c – 7 = -13 ▪ 1.7m2 = 10.625 ▪ 4(z + 5) = 84(b – 1) – 5(b + 1) = 0 ▪ 12 / (x+1) = 21 / (x + 4) Construct linear equations, e.g. The length of a rectangle is three times its width. Its perimeter is 24cm. Find its area.	Show me: • a linear equation with the solution x = 4 • a two step linear equation with the solution x = 4 What is wrong: If 3x– 2 = 2 then 3x = 4 so x = ¾ True/never/sometimes: • Linear equations have one solution • Linear equations can be solved • When solving equations involving brackets, such as 3(x-2) = 12, the first step is to multiply the brackets out Convince me that: • 6 = 2x – 8 has only one solution. • the solution to the equation 3x – 2 = 2 is not x = ¾.
Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT; write an expression to describe the nth term of an arithmetic sequence.	
Generate the first five terms if, e.g. ▪ you start with 100 and subtract 5 each time ▪ you start with 2 and double each time ▪ the nth term is n+3 ▪ the nth term is 105 - 5n ▪ the nth term is 2n - 0.5 Find the nth term for, e.g. ▪ 7, 12, 17, 22, 27, ... ▪ -12, -7, -2, 3, 8, ... ▪ 4, -2, -8, -14, -20, ... Use a spreadsheet to generate tables of values and explore term-to-term and position-to-term linear relationships	Show me: • a sequence that has the term-to-term rule of +2. • the sequence that has the position-to-term rule of +2. • the sequence that has the nth term of i) n+2 ii) 3n+2 Convince me that: • the nth term of the sequence 5, 8, 11, 14, … is 3n+ 2 • that the nth term of the sequence 15, 11, 7, 3, … is 19 – 4n
Plot the graphs of linear functions, where y is given explicitly in terms of x; recognise that equations of the form y = mx+c correspond to straight-line graphs	
Plot the graphs of simple linear functions using all four quadrants by generating co-ordinate pairs or a table of values. e.g. ▪ y = 2x - 3 ▪ y = 5 - 4x Understand the gradient and intercept in y = mx + c, describe similarities and differences of given straight line graphs. e.g. ▪ y = 2x + 4 ▪ y = 2x - 3	Show me: • a linear function that passes through the point (0,3) • a linear function with a gradient of 4 • a linear function with a negative gradient True/never/sometimes: • If you increase the value of m, the line is steeper • If you increase the value of c, the line is steeper

Without drawing the graphs, compare and contrast features of graphs such as: ▪ $y = 3x$ ▪ $y = 3x + 4$ ▪ $y = x + 4$ ▪ $y = x - 2$ ▪ $y = 3x - 2$ ▪ $y = -3x + 4$	Convince me that: • the graph of $y = 2x - 2$ goes through the point $(0,-2)$ • the graph of $2y = 2x + 4$ goes through the point $(0,2)$
Construct functions arising from real-life problems and plot their corresponding graphs; interpret graphs arising from real situations	
The graph below shows information about as race between two animals – the hare (red) and the tortoise (blue)  ▪ Who was ahead after 2 minutes? ▪ What happened at 3 minutes? ▪ At what time did the tortoise draw level with the hare? ▪ Between what times was the tortoise travelling fastest? ▪ By how much distance did the tortoise win the race?	Show me a description of a journey that produces a distance/ time graph with a shape similar to a trapezium. True/never/sometimes: • On a distance/ time graph, if the graph is horizontal then the object is travelling at a constant speed. • On a distance/ time graph, if the graph has a negative gradient then the object is travelling downhill. Convince me that on a distance/ time graph, if the graph is horizontal then the object is stationary.

Shape, Space and Measures

Classify quadrilaterals by their geometric properties	
Know the properties (equal and/or parallel sides, equal angles, right angles, diagonals bisected and/or at right angles, reflection and rotation symmetry) of: ▪ an isosceles trapezium ▪ a parallelogram ▪ a rhombus ▪ a kite ▪ an arrowhead or delta	Show me a quadrilateral: • that has one pair of parallel sides • whose diagonals bisect at right angles • that has two lines of symmetry • that has rotational symmetry of order 2. What is the same/different about: • trapezium, parallelogram, rhombus, kite • rhombus, kite, arrowhead, trapezium Convince me that: a rhombus is a parallelogram but a parallelogram is not necessarily a rhombus. a trapezium can not have three acute angles
Solve geometrical problems using properties of angles, of parallel and intersecting lines, and of triangles and other polygons	
Explain why, e.g. ▪ equilateral triangles, squares and regular hexagons will tessellate on their own, but other regular polygons will not; ▪ squares and regular octagons will tessellate together.	Show me: • a polygon that will tessellate on its own • a pair of polygons that tessellate with each other. True/never/sometimes: • Regular polygons will tessellate on their own • Triangles tessellate on their own • Quadrilaterals tessellate on their own • Hexagons tessellate on their own Convince me that • the sum of the exterior angles of a polygon is 360°. • a regular hexagon will tessellate on its own. • squares and regular octagons will tessellate together.
Identify alternate and corresponding angles; understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360°	
Know the difference between a demonstration and a proof.	Show me a pair of alternate / corresponding angles True/never/sometimes: • The sum of the angles of a triangle is 180º • The sum of the angles of a quadrilateral is 360º. • Alternate angles are equal • The size of a corresponding angle equals the size of an alternate angle. Convince me that • the sum of the angles of a triangle is 180º. • the sum of the angles of a quadrilateral is 360º.
Devise instructions for a computer to generate and transform shapes and paths.	
Draw a square / hexagon / equilateral triangle using LOGO and use the instructions to compare with the exterior angles of a polygon	What is the same/ different about: ▪ Forward 90, Right 90 ▪ Right 90, Forward 90
Visualise and use 2-D representations of 3-D objects	
Visualise solids from an oral description, e.g. Identify the 3-D shape if: ▪ The front and side elevations are both triangles and the plan is a square. ▪ The front and side elevations are both rectangles and the plan is a circle. ▪ The front elevation is a rectangle, the side elevation is a triangle and the plan in a rectangle. Is it possible to slice a cube so that the cross-section is:	Show me: ▪ a solid with a plan that is square. ▪ a solid with front and side elevations both triangles. ▪ a solid with front and side elevations both triangles and the plan is a square.

▪ a rectangle? ▪ a triangle? ▪ a pentagon? ▪ a hexagon?	
Enlarge 2-D shapes, given a centre of enlargement and a positive whole-number scale factor	
Construct an enlargement given the object, centre and scale factor Find the centre and / or scale factor from the object and image	Show me an enlargement. Convince me: ▪ how to enlarge a shape or object with the centre of enlargement outside the shape. ▪ how to enlarge a shape or object with the centre of enlargement inside the shape. ▪ how to find the centre of enlargement given the object and image.
Know that translations, rotations and reflections preserve length and angle and map objects on to congruent images	
Find missing lengths / angles on diagrams that show an object and its image	Show me a i) translation, ii) rotation, iii) reflection What is the same/different about: translation, rotation, reflection, enlargement True/never/sometimes: • Translations preserve length • Rotations preserve length • Reflections preserve length • Enlargements preserve length • Translations map objects onto congruent images • Rotations map objects onto congruent images • Reflections map objects onto congruent images • Enlargements map objects onto congruent images Convince me that: • Translations, rotations and reflections preserve length • Translations, rotations and reflections map objects onto congruent images
Use straight edge and compasses to do standard constructions	
Construct ▪ the mid-point and perpendicular bisector of a line segment ▪ the bisector of an angle ▪ the perpendicular from a point to a line ▪ the perpendicular from a point on a line	Show me: ▪ a construction you can do using a straight edge and a pair of compasses ▪ a construction where it is important to keep the same compass arc. Convince me how to construct: ▪ the mid-point and perpendicular bisector of a line segment ▪ the bisector of an angle ▪ the perpendicular from a point to a line ▪ the perpendicular from a point on a line
Deduce and use formulae for the area of a triangle and parallelogram, and the volume of a cuboid; calculate volumes and surface areas of cuboids	
Suggest possible dimensions for triangles and parallelograms when the area is known Find three cuboids with a volume of 24cm^3 Find three cuboids with a surface area of 60cm^2	Show me: • a triangle with an area of 24cm^2. • a parallelogram with an area of 24cm^2. • a cuboid with a volume of 60cm^3. • a cuboid with a surface area of 60cm^2. True/never/sometimes: • To find the area of a triangle, you multiply the base by the height and half the answer • You can build a solid cuboid using any number of interlocking cubes. Convince me that: • you have to multiply the base by the

	perpendicular height to find the area of a parallelogram. • you have to multiply the base by the perpendicular height and half the answer to find the area of a triangle.
Know and use the formulae for the circumference and area of a circle	
A touring cycle has wheels of diameter 70cm. How many rotations does each wheel make for every 10km travelled? A door is in the shape of a rectangle with a semi-circular arch. The rectangular part is 2m high and the door is 90cm wide. What is the area of the door?	Show me: • a circle with a circumference greater than 50 cm. • a circle with an area greater than $100cm^2$. Convince me that: • the circumference of a circle with radius 10cm is 62.8cm (to 1dp) • the area of a circle with radius 10cm is $314cm^2$ (to 3sf)

Handling Data

Design a survey or experiment to capture the necessary data from one or more sources; design, trial and if necessary refine data collection sheets; construct tables for large discrete and continuous sets of raw data, choosing suitable class intervals; design and use two-way tables	
Investigation of jumping or throwing distances: ▪ Check that the data collection sheet is designed to record all factors that may have a bearing on the distance jumped or thrown, such as age or height. ▪ Decide the degree of accuracy needed for each factor. ▪ Recognise that collecting too much information will slow down the experiment; too little may limit its scope.	Show me an example of a data collection sheet. Show me an example of a class interval. Convince me how you decided on the sample size.
Select, construct and modify, on paper & using ICT: ▪ ***pie charts for categorical data;*** ▪ ***bar charts and frequency diagrams for discrete and continuous data;*** ▪ ***simple time graphs for time series;*** ▪ ***scatter graphs.*** ***Identify which are most useful in the context of the problem***	
Understand that pie charts are mainly suitable for categorical data. They should draw pie charts using ICT and by hand, usually using a calculator to find angles Draw compound bar charts with subcategories Use frequency diagrams for continuous data and know that the divisions between bars should be labelled Know that it can be appropriate to join points on a line graph in order to compare trends over time	Show me set of discrete data. Show me set of continuous data. Convince me the information you need to calculate the size of the angle for each category when drawing a pie chart. When considering a range of graphs representing the same data: ▪ Which is the easiest to interpret? Why? ▪ Which is most helpful in addressing the hypothesis? Why?
Find and record all possible mutually exclusive outcomes for single events and two successive events in a systematic way	
Use a possibility space diagram to show all outcomes when two dice are rolled together	Show me an example of a possibility space. What is wrong: ▪ The probability of getting exactly one head when tossing two coins is ¼ . ▪ The probability of getting two sixes when rolling two fair dice is 1/12 Convince me that the probability of getting exactly one head when tossing two coins is ½. Convince me that the probability of getting two sixes when rolling two fair dice is 1/36.
Know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems	
Two coins are thrown at the same time. There are four possible outcomes: HH, HT, TH, TT. How many possible outcomes are there if: ▪ Three coins are used? ▪ Four coins are used? ▪ Five coins are used?	Show me a pair of mutually exclusive events. True/never/sometimes: • The outcomes of tossing three coins are mutually exclusive • The outcomes of rolling die and tossing a coin are mutually exclusive • The sum of probabilities of all mutually exclusive outcomes is 1 Convince that a pair of outcomes for an experiment are mutually exclusive

	Convince me the sum of sum of probabilities of all mutually exclusive outcomes is 1
Communicate interpretations and results of a statistical survey using selected tables, graphs and diagrams in support	
Using selected tables, graphs and diagrams to support; describe the current incidence of male and female smoking in the UK, using frequency diagrams to demonstrate the peak age groups. Show how the position has changed over the past 20 years; using line graphs. Conclude that the only group of smokers on the increase is females aged 15 -25.	Show me a graph that you could use to communicate the results of a statistical survey. Convince me why you used that i) table ii) graph