

Problem solving with EYFS, Key Stage 1 and Key Stage 2 children

Finding rules and describing patterns

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Introduction

This resource focuses on problems that fall into the category 'Patterns and relationships'. To help children become good problem solvers it is important that they are taught and acquire essential skills and strategies and understand how and when to use them.

When solving 'Patterns and relationships' problems, the strategies children need to be able to draw on include:

- oral rehearsal of the pattern they can see to refine their thoughts
- having a system for recording the pattern, e.g. using pictures, tables or lists of calculations
- organising the recording of patterns, e.g. making an ordered list or table and adapting it as more information is collected in order to predict what comes next
- eventually, describing same general term using mathematical notation even if they see the sequence differently.

There are lessons for each age group across two years, and a selection of activities for the Early Years Foundation Stage, based on one theme.

You should adapt the problems for the needs of your children. For example, in the Year 3/4 problem that involves party bags you could: change the number of presents in the bag; alter the rule that presents can't be the same; change the context.

Throughout the lessons, there are notes to indicate particular features that you might want to consider or look out for when working with children, indicated by the following symbol:



Foundation Stage/Year 1

Repeating patterns

Primary Framework objectives

Reception

- Talk about, recognise and recreate simple patterns
- Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices

Year 1

- Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given conditions
- Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures

Vocabulary

pattern repeat same different predict

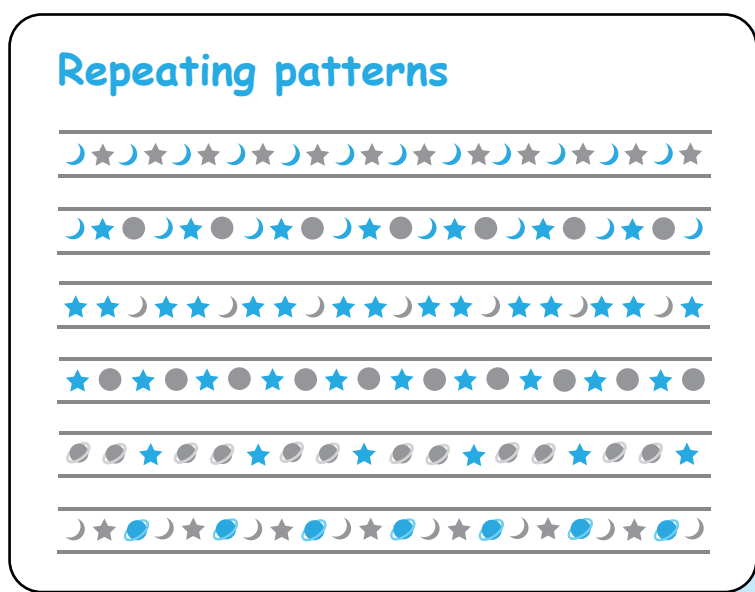
Necessary prior knowledge

Some previous experience of pattern and shape would be helpful

Ordinal language

Resources

- Wallpaper borders with repeating patterns
- Gummed shapes, stamps or photocopied shapes
- Drum or tambourine
- Whiteboards
- Interlocking plastic cubes, bricks, or construction equipment



Activity 1

It will be helpful if the children have resources (stamps, photocopied images, stickers) which they can use later in the activity.

Show the children some pictures or images displayed on an interactive whiteboard, including wallpaper and pictures from nature. Ask children if they know what a pattern is.

Q. What other kinds of patterns are there? Can you see any patterns in the classroom, on the walls or on the floors, around the school? Are there any patterns at home, in the wallpaper, or on your clothes?



It is important to build on children's awareness of pattern in their surroundings. Encourage them to use shape and positional vocabulary to describe them.

Show the children a pattern of objects along a wallpaper border (moon, star, moon, star, etc.)



Allow children enough time to think and talk about their own ideas before discussing with the whole class.



If you have an interactive whiteboard, you could use clip art of moons and stars to copy and paste to continue the pattern.

Q. Describe what you can see to each other.

Q. How many different objects are there? Is there a pattern? What is the pattern? Which object would come next in the pattern?

Discuss which object is first, second, third, etc. Display some of the mathematical vocabulary as the children describe the pattern, e.g. next to.

Q. Which object will be after the last one shown? How can we check that we are right?

Introduce a second pattern with three objects, e.g. moon, star, sun, moon, star, sun, and so on.



Using shapes on an interactive whiteboard or large cut out shapes, ask the children to demonstrate which objects would be next in the pattern.

Q. How many objects are in this pattern? Which is first? Second? Third?

Q. What happens next?

Display some of the mathematical vocabulary as the children describe the pattern, e.g. in between.

Q. What will the next object be? And the next? Why?

Show the children a pattern using two objects where one is repeated, e.g. star, star, moon, star, star, moon, etc.



Q. How many objects are in this pattern? What is first? Second? Third?
And then what happens?

Q. What object will be next? And the next?



Clarify children's understanding of the terms 'same' and 'different/difference'. Encourage the use of them when children are in discussion pairs. Ensure that they see that there are many different possibilities.

Give the children the resources (e.g. stamps, stickers, photocopied shapes) to make their own patterns using the same objects. Ask them to make different patterns to yours. They should discuss what is the same and what is different about their patterns.

Drawing together

Ask children to close their eyes while you cut a border into three pieces.



Encourage children to use mathematical vocabulary about shape and position.

Q. How can we reorder these to create a pattern? What clues can we use?

Ask children to hide part of their pattern while their partners close their eyes.

Q. What's hidden? How do you know?

You could also use a 'slidy box' format or hide-and-reveal facility on an interactive whiteboard.

Activity 2

Think about referring to pattern on an interactive display to support some children when they are discussing pattern generally.

Ask the children to sit in a circle.

Explain that you are going to show the children a pattern, and that they will need to describe what the pattern is. Ask the first child in the circle to stay sitting, the second child to stand up, the third child to stay sitting, the fourth child to stand up, and so on.

Stop at a given point and ask:

Q. What am I going to ask the next child to do? Explain how you know.



Initial assessment opportunities are important here through focused and targeted questioning.

Choose a child within the circle and discuss whether they will be sitting or standing when their turn comes.

The children could also be clapping, clicking fingers, doing arm movements, crossing and uncrossing legs or arms, spinning around, etc.

Q. How do you know?

Start the pattern again at a different point in the circle. Introduce the word 'predict' and encourage the children to predict whether the fifth child in the circle will be sitting or standing. What about the tenth?

Q. How did you work it out? What information did you use?



Encourage children to use mathematical vocabulary about shape and position.

Move the group to a larger space where there is freedom to move. Ask the children to work in groups of four to create a pattern using themselves so that the pattern is two different shapes and then repeated by the other two children.

Invite the groups of children to the front and ask the others to describe the patterns.

Q. Can you go to the fifth position and make yourself into the shape that makes the repeating pattern correct?

Encourage the children to all stand very tall, up on tiptoe, arms, hands and fingers reaching to the sky. Explain that this is the first shape in a pattern. Now ask the children to stretch their arms out and their fingers wide and to crouch down low. Explain that this is the next shape in the pattern.

Ask them to make the first shape, then the second, and so on. Use a drum or a tambourine to tap when you want the children to change to the next position. This will develop listening skills and provide a sense of order to the pattern.

Q. Can anyone think of another shape to add to our pattern?

Encourage one child to demonstrate a shape and another to describe the shape they have made. Emphasise the mathematical vocabulary used.

Now ask all children to form the first shape on the first bang of the drum and hold it for a few seconds. On the second bang children move into position for the next shape, and then finally on the third bang children move into the last shape.

Q. How many different shapes have we created?

Q. Is this a repeated pattern?

Q. How could the pattern become a repeated pattern?



Opportunities for explanation and reasoning here will assess children's understanding of 'repeated patterns'. Allow children time to reflect before they respond.

Take responses from the children.

Q. Can anyone describe to us how our three movements could become a repeated pattern?

Ask pairs to discuss how they could change the three movements into a repeated pattern of movements. Choose one pair to demonstrate.

Q. How many movements did the pair make in total? What was the first shape they made? The second? The third? Then what happened?

Q. How many times did they repeat the first sequence of movements?

Ask children to make up their own sequence of two or three movements and to repeat it, forming a pattern. Ask several pairs to perform their patterns.

Q. How many different shapes were in their pattern? Which was first? Second? Third? What did they do next?

Activity 3

Give each pair or group of children construction equipment or interlocking cubes/bricks of different colours and sizes. Ask the children to look at the individual cubes/bricks and describe them to each other.

Encourage more-able children to use size as well as colour or to have three rather than two different bricks in their patterns.



There is an opportunity for assessment of objective as children engage in a more independent activity.

- Q. Can you create a line of blocks which shows a repeating pattern?
- Q. Can you describe your pattern to somebody else at your table?

If the children find this too easy, ask one child to remove two blocks from different places in the pattern.

Display two coloured cubes in the centre of an interactive whiteboard or on a flip chart. Tell the children that this is the middle of a repeating pattern.

- Q. Can anyone from the group suggest how we might add some cubes to the right or left hand side to continue the pattern?
- Q. How did you use the information to start the pattern?
- Q. Did you have to change your ideas as the pattern was added to?

Activity 4

Use an example of a number sequence forwards and backwards that the children have been practising, e.g. counting in twos from 0 or 1, counting in fives or tens. Using a puppet, ask the children to close their eyes and listen to the puppet saying a number sequence where it misses out one or two numbers. Ask children to use their whiteboards to record the missing numbers in order.



It is helpful for children to link their work on patterns with counting in particular step sizes, so that they see this is one particular kind of pattern.

- Q. Does this pattern continue forwards and backwards?

Ask the children to listen again. This time include a deliberate mistake in the sequence (repeating numbers, reversed numbers, missing numbers), e.g. 10, 20, 30, 50, 40, 60, 70.

- Q. What is wrong with this pattern of numbers?

Ask the children to work in pairs to write their own sequences of numbers with one deliberate mistake. They should ask another pair to spot the mistake.

Year 1/Year 2

Activity 1: Teddy's presents

Objectives

- Describe simple patterns and relationships involving numbers or shapes; decide whether examples satisfy given condition
- Describe ways of solving puzzles and problems, explaining choices and decisions orally or using pictures

By the end of the lesson, children will be able to:

- recognise and describe a simple number sequence;
- solve and explain their solution to a given problem.

Vocabulary

pattern sequence continue predict

Necessary prior knowledge

Counting, knowledge of the number system

Simple addition

More than, less than

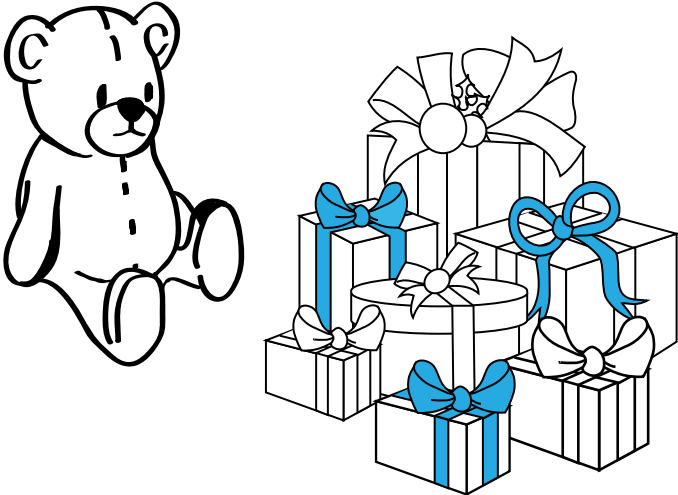
Recognise, say, copy and extend simple patterns

Resources

- Teddy bear and birthday presents (or use clip art on an interactive whiteboard)
- Small cubes or multi-link

Teddy's presents

Teddy is seven years old today.



Teddy has seven presents today.
Each year he is given the same number of presents as his age. How many presents has he had since he was born?

Main teaching activity

Set the scene for the problem by saying that today is Teddy's birthday. Show Teddy with his seven presents.

- Q. How many presents has Teddy got today??
- Q. How do you know?
- Q. How many presents would he have got last year if he got the same number as his age??
- Q. How many will he have next year?
- Q. How many would he have had two years ago?
- Q. How do you know?

If children are unsure, add and remove presents to show the number for each question.

Ensure that children have used language such as one more, one less, same number as how old he is, etc, and write up the vocabulary as it is used. Ask similar questions, ensuring that all children are clear.



Encourage specific vocabulary so that children learn to explain clearly.

- Q. How many presents did Teddy have when he was one?
- Q. How many presents did he have altogether by the time he was two?

Show this by making a pile of one present and another of two presents. Make sure the children understand that Teddy has this year's present but also last year's present and that they need to be added together.

If you are using clip art on an interactive whiteboard, copy and paste the first present and then add one more present.

Discuss how you might record this in a way that is clear to everyone. Suggestions might include using cubes or multi-link, pictures, tallies, or calculations (1, 1 + 2, ...), or a mixture of these. Demonstrate some of these ways of recording for the candles blown out by the time Teddy is two.



Select a few children, whom you have overheard clearly explaining what they are doing, for feedback..

Ask children to work in pairs, choose a method of recording and use this to show the number of presents for Teddy's first, second, third and fourth birthdays.

Drawing together

Choose a variety of recordings, but no more than three, e.g. a picture recording, a cube recording and a calculation recording, and invite children to explain what they have done.

- Q. Which way of recording is clear and helps us to work out what we need to know quickly?
- Q. Which helps us to see how the pattern of the presents grows?
- Q. Could we find out how many presents Teddy will have next year, from these recordings?

Ensure that children are aware of:

- the cumulative nature of the number of presents;
- the need to record in clear columns to see the pattern growing.

Show children how to move from using cubes to using pencil and paper, keeping recording in columns, e.g.

I		1
I	II	3
I	II	III
		6

Differentiation may be necessary for those children who are not secure with counting, or totalling to 28, and for those who find the answer quickly. You may ask some children to find the number of presents received when Teddy is five, or eight.

Now ask the children:

Q. If Teddy is seven years old today, what is the total number of presents he has been given since he was born?

Explain that they won't be able to answer the question straight away.

Ask children to check that their partner understands what the question is asking and to discuss what they will do to solve it. Say that they may change the way they record if they can now see a better way of doing it, or they may stay with their own recording.



This paired discussion gives an opportunity for children to develop their reasoning skills and use of vocabulary.

Allow children time to work on the problem in pairs.



Circulate and observe surprises – children who exceed expectations or don't meet expectations. Be aware of a dominant partner, and target questions at the passive partner to ensure understanding.

As you work with one or more groups, ask children to clearly show their recording for the fourth or fifth birthday and to talk about it using specific language. These may be given on cards, if the child is able to read them, e.g. more, less, altogether, as well as.

If the language used is not concise, ask them to use the specific language and to practise their explanation with someone else.



Practising their explanations with a partner can build confidence and provide you with some exemplar pairs to draw upon in the plenary.

When they can do this, have some prepared questions ready, for example:

- Q.** What if Teddy was eight? Could you solve this without writing or drawing?
- Q.** What language would you use to explain it? What would be the important words?
- Q.** What questions can you ask a partner?

When most children have solved the problem ask them to share their thinking with another pair at their table so that all four understand each other's work.

Plenary

Collect answers to the seventh birthday presents question with the class. If there are different answers, explore why and deal with misconceptions.



Help children to recognise that there is more than one way of recording, but that some ways are clearer or more useful than others.

- Q. Was your recording easy to understand?
- Q. Did anyone change their way of recording because they found a better way?
- Q. Can you explain what was clearer about it?

Look for examples of other clear recordings that show the accumulated facts and a clear column structure.

Discuss patterns that children have noticed. Draw out:

- increasing patterns from recording, i.e. that we include all the years before and then this year's birthday;
- number patterns, e.g. in recording

1 + 2

1 + 2 + 3

Other patterns which children may have noticed are:

- In the solution 1, 3, 6, 10, 15, 21, 28
the differences are 2, 3, 4, 5, 6, 7
i.e. the last number that was added is Teddy's new age.
- Triangle shapes are made by the way in which the presents are arranged, represented by cubes.

I

I II

I II III

I II III IIII

I II III IIII IIII

I II III IIII IIII IIII

I II III IIII IIII IIII IIII

Review how the following recording is easier for some children to add up, and make reference to bonds to 10 if some children noticed this.

1

1 2

1 2 3

1 2 3 4

1 2 3 4 5

1 2 3 4 5 6

1 2 3 4 5 6 7



Now that children understand the problem, they should be able to make an informed guess, or be able to say how we can work it out.

Q. If Teddy was eight, can you predict how many presents he would have received?

For those who have done this, ask how many presents Teddy would have received if he was ten.

Q. How could we test this?

Invite one child to do this.

Q. What were the most useful strategies we used to help us answer the problem?

Q. Can we make a list to help us remember for next time?

- talking about the question to understand what it is asking us
- clear recording
- talking and explaining
- looking for patterns.



It is important that children see that these strategies could be used to solve other problems, not just this one.

Year 2/Year 3

Activity 1: Hopscotch grid

Objectives

- Identify patterns and relationships involving numbers or shapes, and use these to solve problems.
- Describe and explain methods, choices and solutions to puzzles and problems, orally and in writing, using pictures and diagrams.
- Read, write and order whole numbers to at least 1000...; count on from and back to zero in single-digit steps or multiples of ten.

By the end of the lesson, children will be able to:

- describe the rule of a pattern or relationship in own words or pictures;
- test predicted terms to see if a possible rule works;
- use pattern to work out the tenth term in a sequence.

Vocabulary

pattern number sequence increase first, second, tenth, etc.

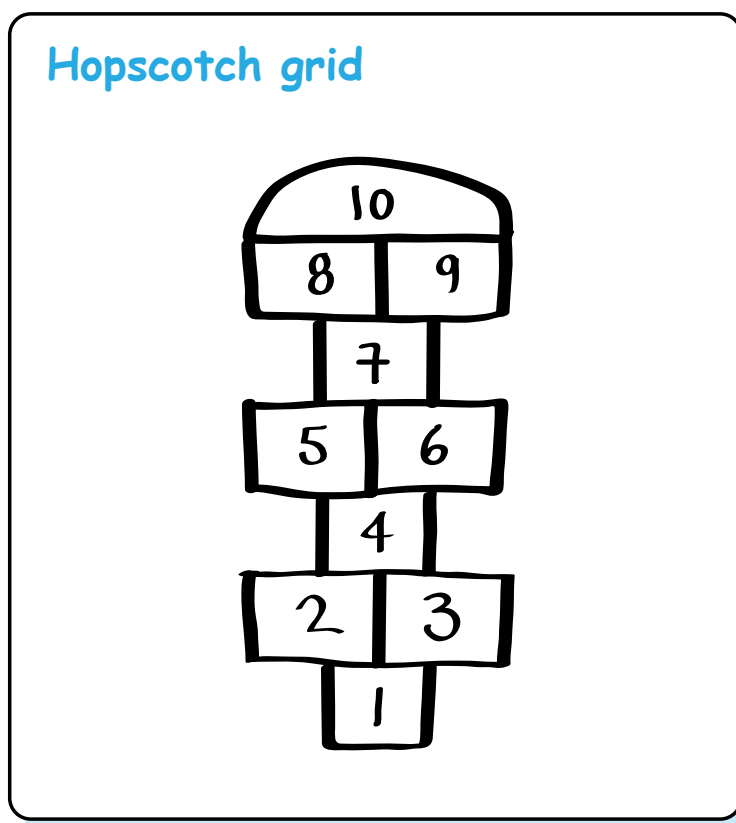
Necessary prior knowledge

Counting in threes and fives

Odd and even numbers

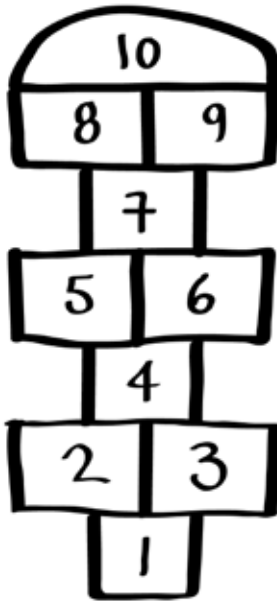
Resources

- Hopscotch grid
- Digit cards



Main teaching activity

Draw a hopscotch grid on the board or display one on an interactive board.



It may be helpful if children have jotters or rough books available.

Now ask the children questions such as the following:

- Start behind 1. Hop, jump, hop. What number are you on?
- Start behind 1. After a sequence of five moves are you on an odd or an even number?
- What happens if you make two more moves in the sequence?
- If we continue the grid pattern beyond 10, will 15 be in a single square or in a pair of squares?
- What is the difference between each of the numbers in the single squares? What is the difference between each number in the right-hand square of the pair of squares?
- What if the starting number was 3? Would the pattern change? What would the number be in the square currently holding 10? And in the square holding 99?

After the initial input ask the children:

Q. What number patterns can you see?

Allow children time to discuss in pairs before taking feedback. Jot down all the number patterns they have spotted. Be prepared for them seeing patterns that you have not spotted yourself!



This paired discussion will involve more children and allow them time to practise explaining their ideas before contributing to the whole-class discussion.

Encourage children to describe the patterns and the rules that they noticed. The sorts of questions you could ask could include:

Q. What is happening to the numbers in the:

- single squares
- left-hand double squares
- right-hand double squares?

- Q. Can you say what the rule is?
- Q. Can you continue the sequence in the:
 - single squares?
 - In the left-hand double square?
 - In the right-hand double square?
- Q. Will the tenth number in the single-square sequence be odd or even?
- Q. Can you predict what the tenth number will be in the right-hand double-square sequence?
- Q. Why is it easier to predict this tenth number than the tenth number in the other two sequences?



The suggested questions are not a definitive list. You will need to pursue questions that build on the patterns and relationships that your children spot.

Encourage children to sketch the grid to test their hypotheses.

Assess children's understanding by asking questions such as 'How do you know?' and 'Why do you think that?' Give them time to explain a range of patterns.

Some children may find it helpful to list the single square numbers in order to help them see the pattern of odd and even numbers.

There is enough material in a grid like this to span more than one lesson.

Ask children to continue working in pairs to discuss, explain and record the patterns they find.

Drawing together

Encourage the children to discuss the strategies that they have used to predict numbers, e.g. 'I knew the sequence was the 3 times table, so I knew the tenth number would be 3×10 ', 'I noticed that the numbers in the second and fourth single squares were even numbers so I knew that the number in the tenth square would be an even number'.

- Q. Would number 14 be in a single square? Would 17 be in a double square? Explain to your partner how you know.



It is important that children have opportunities to rehearse explaining their reasoning orally before they are expected to record their reasoning in writing.

Some children will be able to do this by working with odds and evens and number patterns; others will need to sketch the extension to the grid. Where children have used sketches, encourage them to show them to the rest of the class.

- Q. What if the grid started at 3?

Explore this idea of 'What if ...?' statements with the children and give them time to create some of their own. They can explore grids with different starting numbers.



Here you are giving support to children so that they can ask their own 'What if. . .?' questions.

Plenary

- Q.** Can you design a hopscotch grid made up of single and double squares so that 6, 11 and 16 are in single squares? Convince your partner.



Using questions like these creates good opportunities for you to assess children's understanding of the number patterns they have been working with.

If, after some discussion, children have not spotted that these numbers increase by 5 each time, you could ask them to describe the pattern of numbers, and ask what numbers will need to go in between.

Give children time to rehearse explaining their thoughts to their mathematics partners and then choose some to explain their thoughts to the class.



This rehearsal should help children to clarify thoughts and so improve the quality of their explanations to the class.

Year 3/Year 4

Activity 1: Party bags

Objectives

- Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples
- Describe and explain methods, choices and solutions to puzzles and problems, orally and in writing, using pictures and diagrams

By the end of the lesson, children will be able to:

- recognise, describe and continue a number sequence;
- explain their generalisations.

Vocabulary

- pattern sequence continue predict rule

Necessary prior knowledge

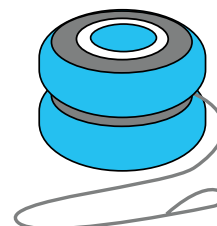
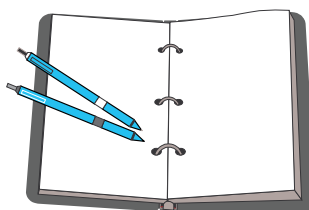
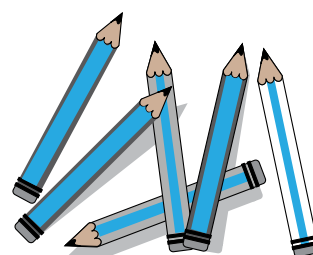
- Counting, knowledge of the number system
- Knowing strategies for adding several numbers
- Familiarity with more than, less than
- Recognise, say, copy and extend simple patterns

Resources

- Presents and bags
- Clip art of similar

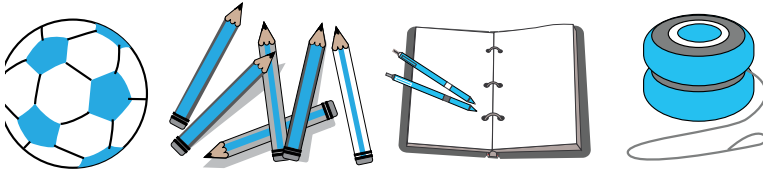
Party bags

Jack is having a birthday party and giving out party bags to his friends.



How many different party bags with two presents in each can be made with 10 different items?

Introduce the problem by saying Jack is having a party. Explain that he is giving out party bags to his friends. Each friend will get two presents and each bag will be different.



Q. If Jack has four presents to choose from, how many different party bags can he make?

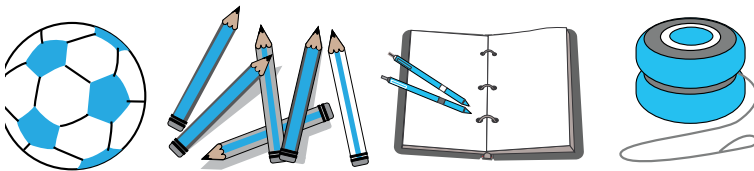
Q. Can you explain your method to your partner?

Explaining to a partner allows children to clarify their thinking.

Q. What is the best way to record to keep track of the different party bags?

Circulate and listen to the children's responses as they begin to consider how they might solve the problem.

Take feedback from the children, recording their method on the board. If it's not suggested, say one possible way to record is using symbols. For example, we can give each of the presents a letter:



A

B

C

D

AB BC CD DE

AC BD CE

AD BE

AE

Ask the children to work with a partner and record how many different party bags there would be if there were five presents and six presents.

Inviting pairs to discuss together provides a good speaking and listening opportunity. It also helps to develop their reasoning skills.

Q. Did you change your method of recording? Why?

Q. How many party bags are there and can you explain your thinking?

Q. How could you display the information for the number of party bags with four, five and six presents?

Circulate and listen to children's predictions and reasoning.

Take feedback and record suggestions including recording the information in a table.

Number of presents	Number of party bags	Total
4	AB BC CD AC BD AD	6
5	AB BC CD DE AC BD CE AD BE AE	10
6	AB BC CD DE EF AC BD CE DF AD BE CF AE BF AF	15

- Q. How would we add the information for three party bags to the table?
- Q. Can you predict how many party bags there would be for seven presents?

Ask the children to add the information to the table.

Number of presents	Number of party bags	Total
3	AB BC AC	3
4	AB BC CD AC BD AD	6
5	AB BC CD DE AC BD CE AD BE AE	10
6	AB BC CD DE EF AC BD CE DF AD BE CF AE BF AF	15
7	AB BC CD DE EF FG AC BD CE DF EG AD BE CF DG AE BF CG AF BG AG	21

- Q. What patterns do you notice? Ask the children to feedback on all the patterns they can see.

Drawing together

Q. Can we use the information in the table to work out how many party bags there would be if we had 11 different presents?

Agree we can add on to the total – the number added increases by one each time so:

$$21 + 7 + 8 + 9 + 10$$

Or we can see from the number of party bags displayed each time we can add:

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$$

Q. Is there a quick way to add these numbers?

Give the children a few moments to discuss this.

Agree that looking for pairs that make 10 is an efficient method.

$$10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

This provides a useful opportunity to remind children to identify and use efficient methods of calculation.

Model recording to show that four pairs make 10, and then we add 5, and then add 10, making a total of 55. Make sure the children understand that this means Jack has made 55 party bags and therefore the problem has been solved.

Q. What if we were working out how many party bags Jack made with 16 presents? What numbers would we need to add?

Q. Would we still look for pairs that make 10? What would pairs of numbers need to total?

Establish that we could look for pairs that make 15.

$$14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$$

Plenary

Q. What did we do to work out how many party bags we had?

Q. Would this method always work?

Q. Using what we have learned today, can you create a rule?

Here, the children are learning to produce a generalised statement.

Tell children that their rule should begin with this phrase: 'To find the total number of party bags you ...' and include some or all of these words: total, add, altogether, age, previous.

Ask children to work in pairs to agree a rule.

Invite several children to share their rules and the rest of the class to evaluate the clarity of the rules shared.

Year 3/Year 4

Activity 2: L-shaped models

Objectives

- Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples
- Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols
- Recognise and continue number sequences formed by counting on or back in steps of constant size

By the end of the lesson, children will be able to:

- describe the rule of a pattern;
- predict the tenth term in a pattern.

Vocabulary

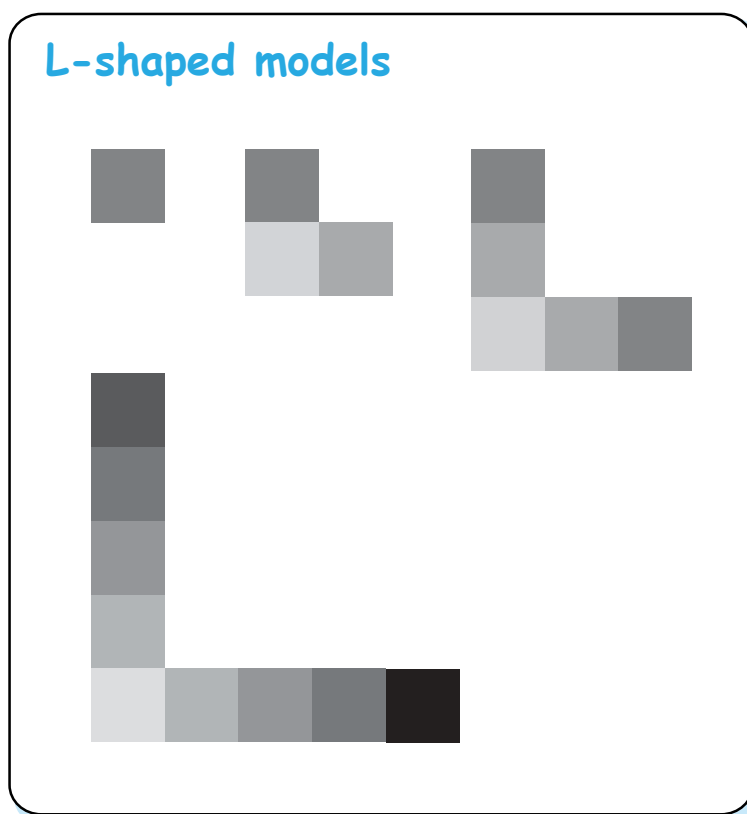
pattern repeating pattern same different
increase bigger first, second, etc.

Necessary prior knowledge

Some experience of recognising and describing patterns

Resources

- Interlocking plastic cubes
- Squared paper



Main teaching activity

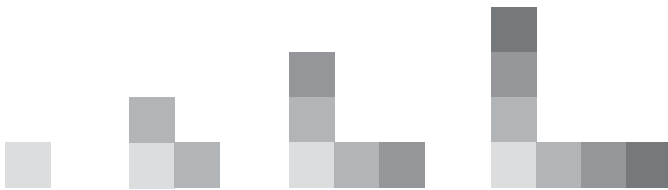
Arrange the children so they can work in pairs, with individual whiteboards or jotters.



How is paired work supporting the children?

Circulate around the class and observe how the children approach the task.

Discuss with the class the variety of patterns and the different ways of representing them, drawing from their previous experiences. Now display some L shapes made from interlocking plastic cubes or use shapes on an interactive whiteboard:



Q. What might the next L shape in the sequence look like?

Ask children to discuss this in pairs, come to an agreement and make the next L shape.

Q. How many cubes might be in the next L shape?

Q. How are the L shapes growing each time?

Ask the children to work with their partner and use the squared paper to record what is happening to the L shape each time. Remind them of the different ways they recorded the clapping patterns, using symbols, shapes and numbers.



Rehearsing descriptions/explanations in pairs involves more children and gives them the opportunity to improve their vocabulary. It also can help them feel more confident in front of the class.

Make sure that there are enough cubes available for the children and some squared paper for recording.

Q. How can you record the way the L shape grows, using numbers?

Q. Would it be useful to create a table?

Q. Would it be helpful to draw the L shape each time?



While children are engaged in this activity look for examples that show different ways of working to use in the plenary.

Ask children to continue finding the number of cubes in the sequence of L shapes. Encourage them to predict the number before making each shape.

Q. Can you predict the number of cubes in the tenth L shape? What about the fifteenth shape? The twenty-first shape?

Q. How can you work out what the 50th shape looks like without drawing out the L shapes?

Plenary

Use children's work to look at the number pattern that has been created.

- Q. What patterns can you see in the numbers you have written down?
- Q. Can you explain to your partner what the rule is?

The children should have spotted that the pattern is the sequence of odd numbers.

Look at the sequence of odd numbers on a number line; ensure that children can articulate that the sequence starts at 1 and increases by two each time.

- Q. Do you need to make or draw the L shape to predict the next number in the sequence?
- Q. Can you and your partner make a shape that would grow in the sequence of even numbers?

Allow children time to discuss and investigate.

You could challenge the children to investigate other letter shapes such as T and E in this lesson or in subsequent lessons. Children could use cubes or squared paper and record their results in tables.

- Q. Can you and your partner explain to each other why you must start with an even number of cubes?
- Q. Can you use what you have learned today to predict or work out how many cubes there would be in the tenth shape?

Agree that if the pattern is the sequence of odd numbers, then the tenth shape will have the tenth odd number of cubes.

Draw the following table on the board:

Shape	1	2	3	4	5	6	7	8	9	10
Number of cubes	2	4	6	8	10	12				?

- Q. This pattern is the sequence of even numbers. Is there a quicker way of finding the number of cubes in the tenth shape than counting in twos?

Encourage children to see the relationship between the number of the shape and the number of cubes in it.

- Q. Can they predict what the twentieth shape would be? Would it be easier to draw the shape or to use the table? Why?

Year 5/Year 6

Activity 1: The candle problem

Objective

- Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of c pens at 15 pence each is $15c$ pence)
- Explain reasoning and conclusions, using words, symbols or diagrams as appropriate

By the end of the lesson, children will be able to:

- express the general term in words and begin to do this algebraically.

Vocabulary

pattern sequence term predict generalise

Necessary prior knowledge

Recognise and extend number sequences

Some experience of relating a term to its position in the sequence

Know the relationship between the areas of right-angled isosceles triangles and squares

Resources

- A3 paper for paper mat activity
- Squared paper available, if required

The candle problem

Mrs Gillespie is 73 today. She has had a birthday cake and candles every year since she was born.



How many candles has she blown out in her lifetime?

Main teaching activity

Set the context of the problem. Mrs Gillespie is 73 today. She has had a birthday cake every year since she was born. How many candles has she blown out in her lifetime?

Ask children to work with a partner and discuss what their understanding of the problem is and what kind of recording may be useful. Remind children that recording needs to be systematic and easy to refer to.



Circulate and listen for discussion demonstrating efficient ways of recording.

Drawing together

Establish that everyone understands the problem and that everyone is clear about the method of recording they will use.

If there are several children who do not know how to record, ask a child whom you have overheard to explain their method.

Work with the uncertain children to begin with until they are secure in recording.

Allow children to explore and enjoy the problem for a short while. It is likely that the children will see that they could add $1 + 2 + 3 \dots 72 + 73$, which will take a long time to calculate.



When children begin to realise that this will take a very long time, draw them together again.

Drawing together

Q. What problems have you come up against?

Discuss problems children have experienced. Pick up on the fact that finding a solution could take a very long time.

Q. What can we do so that we don't have to write down all the numbers of candles and add them up?

Ask children to take a few minutes to discuss possibilities and establish that we need to develop a rule that will get us to the answer more quickly.

Say that you are going to go back to an arrangement of the candles to see what patterns and information can be gained from it. Say that simplifying a problem by using smaller numbers can often help us to understand how we can solve it, perhaps seeing patterns that we don't see when using larger numbers.

Suggest that the candles could be drawn like this:

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|  
| |  
| | |  
| | | |
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This represents the candles blown out in four years.



Circulate and select a group which is moving towards using square numbers. Prepare them to show their thinking to the class before drawing together.

- Q.** Can you see patterns in:
- the way the diagram grows?
 - the shape it makes on the paper?
 - the numbers generated?
 - the number of rows and columns?

Say that these may be things children want to consider.

You could use a 'place mat' activity to generate discussion and creativity in looking for useful patterns/relationships.

Put the first four rows in the centre of the mat. Divide the mat into four.

Each child has a few minutes to jot down anything they notice in their space. They then share with a partner and refine their ideas, discarding anything that will not be useful. Each pair then shares with the other pair until they come up with a group consensus.

Drawing together

Ask the chosen children to describe what they see. Draw out:

- the number of rows and the number of columns are the same;
- this number is Mrs Gillespie's age, i.e. the number of candles on the most recent cake, the number of candles added to the previous total;
- the shape formed by the candles is a right-angled isosceles triangle (it looks like half of a square).

Say that when Mrs Gillespie was five she would have blown out the following number of candles in her lifetime:

```
| . . . .  
| | . . .  
| | | . .  
| | | | .  
| | | | |
```



Here we are showing that a diagrammatic recording of the candles can help us to see and explain a pattern rather than always only looking at the numbers.

Tell the children that numbers in a sequence like this are called triangular numbers, for an obvious reason.

Check that the children understand.

Discuss the arrangement of candles and the square formed by the triangle and empty spaces.

- Q.** What can you say about the number of candles in the triangle, and the number if the whole square was filled?

Agree that the number of candles is a bit more than half the area of the square.

- Q. Will this be the case for the fifth birthday? The sixth? The tenth? The 100th? Always?
- Q. How many candles roughly would Mrs Gillespie have blown out by the time she was six?

Discuss that this would be a bit more than half of 6×6 , i.e. a bit more than 18.

- Q. How many more than 18?

Encourage children to draw the diagram. Agree that the answer is three more than 18, i.e. 21.

Ask the children to discuss how we might work out what to add to half of the square number, i.e. $\frac{1}{2}(n \times n)$ where $n = \text{Mrs Gillespie's age}$.

Drawing together

Children may see and explain this in different ways. Ask at least one group to explain their thinking. For example, some may see that halving the square halves the number in the diagonal, but we want the whole diagonal. Draw out that the bit we need to add on to half the square is half the diagonal.



Describing what they see can help children towards a generalisation.

Circulate while they do this and collect children's ideas. It will be helpful if they rehearse their explanations with their group/partner/you before sharing with the class.

- Q. How can we work out what half the diagonal is?

Refer back to the diagram for the fourth birthday.

| . . .

| | . . Agree that in this case it is 2.

| | | .

| | | |

- Q. What is the number in the whole diagonal?

Agree that the number in the whole diagonal is the number of candles on the most recent cake, i.e. n .

- Q. What is the number of candles?

Agree that this is $\frac{1}{2}(n \times n) + \frac{1}{2}n$.

This can also be written as $\frac{1}{2}(n^2 + n)$.

- Q. What should we do now?

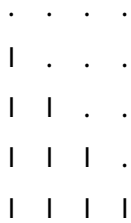
Agree that it would be sensible to check that this works for the next few birthdays. Ask the children to work in pairs to do this.



This check helps children to understand and so be convinced.

- Q. Are you convinced that this will always work?

Alternatively, children may see the pattern in a different way. Some may see and use the lack of symmetry in the square. Some may say that an empty row should be added to the top, as shown below.



Half of this new shape gives the correct number of candles.

Q. If n is the number of candles on the most recent cake, how can we think of the total number?

Agree that we could add n to $(n \times n)$ and then halve this total:

$$\frac{1}{2} (n^2 + n).$$

[This could also be described as $\frac{1}{2} \times n \times (n + 1) = \frac{1}{2} (n^2 + n)$.]

Ask children to check this for several terms.

If you have shown two ways of looking at this, ensure that children realise that different ways of looking at the pattern have resulted in the same expression.

Ask children to solve the original problem, using a calculator if they wish.

Plenary

Ask children to share the answer:

$$\frac{1}{2} (73^2 + 73)$$

$$= \frac{1}{2} \times 5402$$

$$= 2701.$$

Ensure that children realise that this expression $[\frac{1}{2} (n^2 + n)]$ will work for all birthdays.

Q. If Mrs Gillespie lived to be 100, how could we work out how many candles she would have blown out?

Help the children to reflect on how we arrived at a generalisation. You may wish to draw out some or all of the following:

- being sure what the problem was;
- using diagrams to show what was happening;
- predicting and testing;
- explaining what we saw;
- recording this using mathematical language.

Year 5/Year 6

Activity 2: Sequence of models

Objective

- Identify and use patterns, relationships and properties of numbers or shapes; investigate a statement involving numbers and test it with examples

By the end of the lesson, children will be able to:

- accurately predict a later term in a pattern or sequence;
- express the general term in words and begin to do this algebraically.

Vocabulary

pattern sequence term difference general term predict

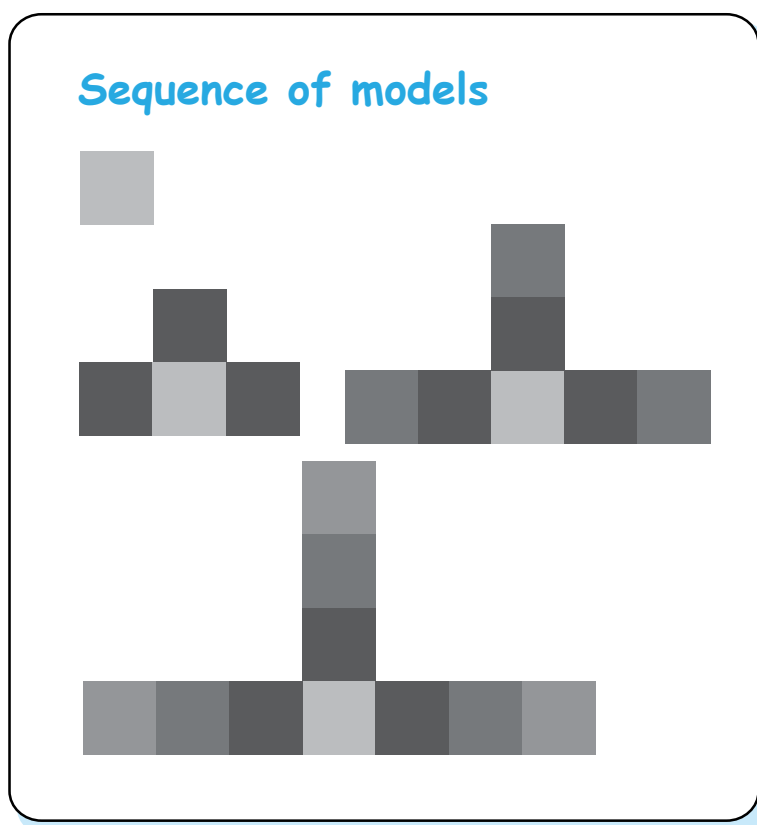
Necessary prior knowledge

Experience of describing and extending patterns

Some experience of recognising the relationship between a term and its position in a sequence

Resources

- Individual whiteboards
- Interlocking plastic cubes
- Squared paper

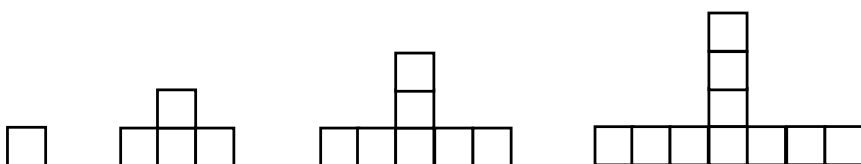


Main teaching activity

Introduce the purpose of the lesson: to describe in words and symbols the general term in a shape pattern.

Show the children these four models made of interlocking plastic cubes in a pattern, ready made or displayed on an interactive whiteboard.

Children who are working significantly behind their year group may benefit from the Year 3/Year 4 lesson, which uses a different pattern of interlocking plastic cubes.



Ask children to build their own four models.



It is important that children do not see a 2-D representation until later in the lesson. Making the models helps children to see how each is different.

Q. How is the pattern developing?

Ask children to explain to their partners how they see it developing.

Take class feedback.

Q. So how exactly do you get from one to the next?

Ask children to discuss this in pairs. Encourage children to use precise mathematical language. Display the language.

Q. Do you both see it developing in the same way? Can there be more than one way of seeing it?

Q. So can you now make the fifth in the sequence, the fifth term? Show what you have made to the person next to you. Have you made the same one? Did you do it in the same way? How did you know what to do?

Q. Could you make the seventh term? Compare yours with the person next to you. How did you do it? Was it harder than making the fifth term?

Highlight how the seventh term is harder because it is not the next term.

Q. Would you have an idea now how to make the twelfth term?

Ask children to discuss this in pairs.



This paired discussion helps children understand the problem and see it in different ways.

Q. How could we record our pattern on squared paper so that we show the way that the pattern is developing?



It is important here to help children make the transition from the concrete to the visual recording on paper.

Draw out, using colour or shading. This is one possible way of doing it.



Discuss strategies children are using to show how they see the pattern developing.

Q. Let's discuss the number of cubes in each part of the pattern. This will help us with devising a general term. What do we mean by a general term?

Encourage children to come up with a definition in pairs. Let one in each pair record on whiteboard.

Agree on a class definition.



Observe how the paired work supports the children's development of the concept.

Suggest that a table will help us organise our thoughts.

Term	1st	2nd	3rd	4th	5th
Number of cubes	1	4	7	10	13
How pattern develops					

Invite children to show you on whiteboards how they see the pattern developing with the numbers.

Here are some suggestions of the ways that children might see the pattern developing.

Term	1st	2nd	3rd	4th	5th
Number of cubes	1	4	7	10	13
Explanation A	1	1 (previous number) + 3 (new ones)	4 + 3	7 + 3	10 + 3
Explanation B	1	3 (horizontal) + 1 (on top)	5 + 2	7 + 3	9 + 4
Explanation C	1	1 + 3 (round the sides)	1 + 6	1 + 9	1 + 12
Explanation D	1	1 + 3	1 + 3 + 3 (three new ones)	1 + 3 + 3 + 3	1 + 3 + 3 + 3 + 3

This may need more work if children have not met the use of n before.

Discuss with children that Explanation A, where the next term is connected to the one before, doesn't help us to find a general term. It only helps us to find the next one if we know the previous one.

Stress that we want to find a way of finding any term if all we know is its position in the pattern.

This assumes that children are familiar with multiplying out brackets.

Explanation B looks at the horizontal and vertical cubes – does this help connect to the position of the model in the pattern?

Q. How could Explanation C be developed? How do we know how many threes to add?

Agree that this is the term number less 1, i.e. $n - 1$.



It is important that children see that the number of cubes connected to position in the pattern is vital to find the general term.

Discuss Explanation D.

Q. Can you see how the number of cubes in the model connects to the term number?

Always start with one and add 'one less lots of three' than the term number.

If we call the term number n then this could be written as $1 + 3 \times (n - 1)$ or $1 + 3n - 3 = 3n - 2$.

Q. Does this work? What happens if the term number is 3? 4? 5? Do we get the number of cubes that we would expect?

Children who use more of the numbers may describe the general term as $3n - 2$. Draw out that this is the same and we can still use the diagrams to explain it, because there are n groups of cubes, but one group only has one cube, not three, so we subtract two.

Plenary

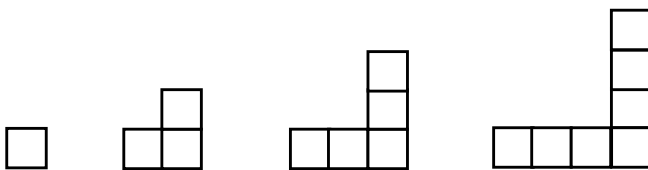
Try a different pattern and ask children to work out what the general term is.

Use the L shape. Draw the pattern on the board.

This pattern is used in the Year 3/Year 4 lesson.

Q. How do we see the pattern developing?

Ask children to discuss this in pairs.



Take feedback.

Q. If we want to find a general term what do we need to do next?

Put the term numbers on the board.

Q. What does the grid look like?

Ask children to discuss, then draw ideas on the board as a class.

Shape	1st	2nd	3rd	4th
Number of cubes	1	3	5	7
Pattern				



This plenary provides an opportunity to see how much the children have grasped.

Q. Can we see how the pattern develops connected to the term number?

Ask children to show their answers on their whiteboards.

Invite children with different suggestions on their whiteboards to come to the front and hold them up for the class to discuss.

Q. Which do we think is most helpful to find the general term?

Q. Why do we think this is the most helpful? Convince your partner.

Shape	1st	2nd	3rd	4th
Number of cubes	1	3	5	7
Pattern	1	1 + 2	1 + 2 + 2	1 + 2 + 2 + 2

Discuss ways in which children convinced their partners.

Q. Can we write the general term using the letter n for the position in the pattern?

Ask children to show their answers on their whiteboards.

$$1 + 2(n - 1) \text{ or } 2n - 1$$

Q. Does your answer work if n is 4, i.e. looking at the fourth term?

Ask children to tell the person next to them what they have learned today.

Draw out that to describe a general term, we need to relate it to its position in the sequence, not just to the previous term.

Although we may see the sequence differently we will end up using the same general term using mathematical notation.

Audience: EYFS, KS1 and KS2 teachers, mathematics subject leaders, LA mathematics consultants.

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